

A String-theory Calculation of Hawking Radiation

การคำนวณการแผ่รังสีฮอว์คิงในทฤษฎีสตริง

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ABSTRACT

A study of black hole thermodynamics show that black holes are radiating. String theory, the promising theory of quantum gravity, should give us how the black holes radiate. We study D1-D5-brane model which closed strings are caused by scattering of open strings living on brane. The decay rate of D1-D5-brane is found to agree with classical Hawking radiation of corresponding 5-dimensional black hole in leading order of low energy approximation.

บทคัดย่อ

จากการศึกษาอุณหพลศาสตร์ของหลุมดำนั้นพบว่าหลุมดำสามารถแผ่รังสีออกมาได้ ทฤษฎีสตริงซึ่งเป็นทฤษฎีที่เชื่อว่าจะสามารถอธิบายแรงโน้มถ่วงในเชิงควอนตัมได้ควรจะช่วยให้คำอธิบายเกี่ยวกับหลุมดำซึ่งสามารถอธิบายปัญหาในข้างต้นได้ เราจะทำการศึกษาแบบจำลองหลุมดำแบบ D1D5 ในทฤษฎีสตริง โดยในแบบจำลองจะมีการแผ่รังสีจากสถานะกระตุ้นของเบรน ที่เกิดจากสตริงแบบปลายเปิดบนเบรนเกิดการชนกันและปลดปล่อยสตริงแบบวงปิดออกมานอกเบรน ซึ่งอัตราการปลดปล่อยรังสีที่ได้จากการแบบจำลองนี้ให้ผลที่สอดคล้องกับการแผ่รังสีฮอว์คิงสำหรับหลุมดำ 5 มิติในระดับการประมาณพลังงานต่ำ

Key Words: Brane decay, Hawking radiation, Black holes in string theory

คำสำคัญ: การสลายตัวของเบรน การแผ่รังสีฮอว์คิง หลุมดำในทฤษฎีสตริง

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1. Introduction

Black holes in the classical theory cannot emit particles. The quantum mechanical effects, however, cause black holes to radiate particles as a thermal black body radiation with temperature $T = \frac{\kappa}{2\pi}$. This is identical with black hole mechanics discovered by James Bardeen, Brandon Carter and Stephen Hawking (Hawking, 1975; Bardeen, Carter and Hawking, 1973).

Recent studies of black hole thermodynamics have been done by using D-branes as a model building. There are some configurations which is BPS saturated brane agree with extremal black hole (Das and Mathur, 1996). This implies that there must exist string states correspond with Hawking-Beckenstein entropy. In the other words, there are some black branes with thermodynamical properties agree with black hole mechanics.

In this paper, we review some calculations and configurations to show the coinciding between D-brane decay and classical black hole radiation.

2. Black Hole Classical Absorption Rate of Low Energy Scalars

In this section, we compute the black hole absorption probability of low energy scalars by using quantum field theory in curved spacetime. The basic idea of semiclassical principle is to treat the matter fields quantum mechanically and the gravity as a background. For our analysis, we will focus on massless scalar field ϕ that satisfy the wave equation,

$$\nabla^a \nabla_a \phi = 0, \quad (1)$$

where g^{ab} is the metric inverse from line element $ds^2 = g_{ab} dx^a dx^b$ and we work in units with $G = c = \hbar = 1$ as same as the most papers. As a

scalar field could be a quantum operator, it must obey the canonical equal time commutation relations, $[\phi(\mathbf{x}, t), \dot{\phi}(\mathbf{x}', t)] = \delta(\mathbf{x} - \mathbf{x}')$. This leads us to write a field in a form of mode expansion,

$$\phi = \int d\omega (a_\omega f_\omega + a_\omega^\dagger f_\omega^*), \quad (2)$$

A set $\{f_\omega, f_\omega^*\}$ is a complete orthonormal set of basis functions. The standard choice of basis functions for scalar field is

$$f_\omega = \frac{1}{\sqrt{2\omega}} e^{-i(\omega t - \mathbf{k} \cdot \mathbf{x})}, \quad (3)$$

where $\omega = +\sqrt{\mathbf{k} \cdot \mathbf{k}}$. The canonical commutation relations for the scalar field imply commutation relations for mode operators $a_\omega, a_\omega^\dagger$, $[a_\omega, a_\omega^\dagger] = \delta(\omega - \omega'), [a_\omega, a_{\omega'}] = 0 = [a_\omega^\dagger, a_{\omega'}^\dagger]$.

(4)

Usually, the vacuum state is defined as the state with the lowest possible energy state. On the other hand, it is a state in the absence of particles which is annihilated by all the annihilation operators a_ω ,

$$a_\omega |0\rangle_a = 0 \quad (5)$$

for all $\omega > 0$. The Fock space of state is constructed by applying creation operators to a vacuum state, for instant, the state $(a_\omega^\dagger)^n |0\rangle_a$ contains n particles with energy ω . By defining a number operator $N_\omega = a_\omega^\dagger a_\omega$ for each mode, so that ${}_a\langle 0 | (a_\omega)^\dagger N_\omega (a_\omega^\dagger)^n | 0 \rangle_a = n$

One can define a second expansion on another complete set of basis $\{p_\omega, p_\omega^*\}$,

$$\phi = \int d\omega' (b_{\omega'} p_{\omega'} + b_{\omega'}^\dagger p_{\omega'}^*), \quad (6)$$

where mode coefficients $b_{\omega'}, b_{\omega'}^\dagger$ also satisfy commutation relations. The annihilation operators $b_{\omega'}$ define another vacuum state, $b_{\omega'} |0\rangle_b = 0$, for all $\omega' > 0$. The number operator for mode in b -states is

$N_{\omega'} = b_{\omega'}^{\dagger} b_{\omega'}$. The creation operators $b_{\omega'}$ also span another Fock space by applying to $|0\rangle_b$.

The scalar field could be expanded with difference sets of basis function. However, one may find that the basis functions f_{ω} and p_{ω} are related to each other through the linear transformation, so-called Bogoliubov transformation,

$$p_{\omega'} = \int d\omega (\alpha_{\omega\omega'} f_{\omega} + \beta_{\omega\omega'} f_{\omega}^*)$$

$$f_{\omega} = \int d\omega' (\alpha_{\omega\omega'}^* p_{\omega'} - \beta_{\omega\omega'} p_{\omega'}^*). \quad (7)$$

In order to satisfy the orthonormality of the basis functions, Bogoliubov coefficients $\alpha_{\omega\omega'}$, $\beta_{\omega\omega'}$ have to follow the equation

$$\int d\omega' (|\alpha_{\omega\omega'}|^2 - |\beta_{\omega\omega'}|^2) = \delta(\omega - \omega'). \quad (8)$$

Those above lead us to the relation between mode coefficients,

$$b_{\omega'} = \int d\omega (\alpha_{\omega\omega'}^* a_{\omega} - \beta_{\omega\omega'}^* a_{\omega}^{\dagger}). \quad (9)$$

So we can now evaluate the expression for $N_{\omega'}$ in the a -vacuum state,

$${}_a\langle 0 | N_{\omega'} | 0 \rangle_a = {}_a\langle 0 | b_{\omega'}^{\dagger} b_{\omega'} | 0 \rangle_a = \int d\omega |\beta_{\omega\omega'}|^2. \quad (10)$$

For the simplicity of further calculation, we consider 1+1 dimensional Schwarzschild metric which is spherically symmetric and stationary black hole solution to Einstein equation.

$$ds^2 = V(r)dt^2 - V(r)^{-1}dr^2, \quad V(r) \equiv 1 - \frac{2M}{r}. \quad (11)$$

The horizon in this coordinates is at $r = 2M$. One might find it is convenient to introduce the tortoise coordinates which bring the metric to a conformally flat form,

$$ds^2 = \left(1 - \frac{2M}{r}\right) [dt^2 - dr^{*2}]$$

$$dr^* \equiv V(r)^{-1}dr$$

$$r^* = r - 2M + 2M \ln\left(\frac{r}{2M} - 1\right). \quad (12)$$

The coordinates (t, r^*) are defined only for $r > 2M$ and asymptotically flat when $r \rightarrow \infty$. In the lightcone coordinates $u \equiv t - r^*$ and $v \equiv t + r^*$, the metric becomes

$$ds^2 = \left(1 - \frac{2M}{r}\right) du dv. \quad (13)$$

In contrast to the Schwarzschild metric which has a coordinate singularity at $r = 2M$, an observer freely falling into the black hole would see a finite curved space while crossing the horizon. We need a suitable coordinate system which is Kruskal coordinates,

$$\bar{u} = -4M \exp\left(-\frac{u}{4M}\right), \quad \bar{v} = 4M \exp\left(\frac{v}{4M}\right). \quad (14)$$

The metric (13) becomes

$$ds^2 = \frac{2M}{r} \exp\left(1 - \frac{2M}{r}\right) d\bar{u} d\bar{v}. \quad (15)$$

With $r = 2M$, the metric (15) becomes $ds^2 = d\bar{u} d\bar{v}$ which is same as Minkowski metric. Hence the freely falling observer crosses the horizon line without seeing singularity.

In the calculation, we use the fact that particle definitions are different among observers. We define a vacuum for observers in free-falling, $|0\rangle_K$ and a vacuum for observers at constant r , $|0\rangle_T$. By comparing the complete basis functions with different coordinates $\{u, v\}$ and $\{\bar{u}, \bar{v}\}$, the number of particle created by black hole is

$${}_K\langle 0|N_\omega|0\rangle_K = \int d\omega' |\beta_{\omega\omega'}|^2 = \frac{1}{e^{4\pi M\omega} - 1} \delta(0). \quad (16)$$

The number density n_ω which $\langle N_\omega \rangle = n_\omega \delta(0)$ is in the form of Bose-Einstein distribution $n_E = \frac{1}{\frac{E}{T} - 1}$, we found that the black hole has a

thermal blackbody radiation with the temperature $T = \frac{1}{4\pi M}$. For other black holes the temperature is

$T = \frac{\kappa}{2\pi}$ where κ is surface gravity of the black hole. The temperature is known as Hawking temperature.

3. 5-dimensional Reissner-Nordstrom Black Hole from Supergravity Solution

For classical radiation of black hole with geometry corresponding to the charges carried by D-branes, we use 5-dimensional extremal black hole metric from SUGRA solution,

$$ds^2 = -f^{-\frac{2}{3}}(r)dt^2 + f^{\frac{1}{3}}(r)dr^2 + f^{\frac{1}{3}}(r)r^2 d\Omega_3^2 \quad (17)$$

where
$$f(r) = (1 + \frac{Q_1}{r^2})(1 + \frac{Q_2}{r^2})(1 + \frac{Q_3}{r^2}). \quad (18)$$

Consider a spherically symmetric massless minimally coupled scalar wave function with higher

angular momentum component (which is not absorbed in low ω limit),

$$\phi(r, t) = R(r)e^{-i\omega t}. \quad (19)$$

The wave equation is now reduced to

$$\left[\frac{d^2}{dr^2} + \omega^2 f(r) - \frac{3}{4r^2} \right] \psi(r) = 0 \quad (20)$$

where $\psi(r) = r^{3/2} R(r)$.

The n -dimensional wave function with $n > 2$ can be decomposed into spherical harmonic wave function. In this case, we consider spherically symmetric mode, so the wave equation becomes

$$W^{(n)} \phi(r) = (W^{(2)} + W^{(n-2)}) \phi(r) \quad (21)$$

$$= \left(\frac{d^2}{dr^2} + \omega^2 f(r) + V^{(n-2)}(r) \right) \phi(r) \quad (22)$$

which $V(r)$ is considered as a barrier-like potential from spacetime geometry. Therefore a particle escaping the black hole needs to tunnel through the potential. This decreases the intensity of the wave by gray body factor, $\Gamma_{gb}(E) < 1$.

$$\Gamma_\omega = \frac{\text{total out going flux at infinity}}{\text{total flux created at horizon}} \quad (23)$$

In order to find a gray body factor, we use matching condition method to find a transmission rate and use low energy limit, $Q\omega^2 \ll 1$, for approximation.

In the calculation, we match boundaries of the solutions across three regions (Das and Mathur, 1996).

i.) Outer region $r \gg Q_i^{1/2}$ In this region we got

$$f \approx 1 + \frac{Q}{r^2} \text{ where } Q = Q_1 + Q_2 + Q_3$$

ii.) Intermediate region $r : Q_i^{1/2}$

iii.) Near horizon region $r \ll Q_i^{1/2}$

By matching these conditions, one must find an absorption probability (equivalent to transmission probability),

$$|A|^2 = \frac{1}{2} \pi \omega^3 \sqrt{Q_1 Q_2 Q_3} = \frac{1}{4\pi} \omega^3 A_H. \quad (24)$$

Similar to the previous section, number density of particle emitted by 5-dimensional black hole is in the form of Bose-Einstein distribution but with a gray body factor. Therefore a total energy emission rate in energy range $(\omega, \omega + d\omega)$ should be,

$$\left[\frac{dE(\omega)}{dt} \right]_{SC} = \frac{d\omega}{2\pi} \frac{\omega \Gamma_{\omega}}{e^{\beta_H \omega} - 1}. \quad (25)$$

In low energy case, gray body factor is approximately to the absorption probability, $\Gamma_{\omega} = |A|^2(\omega)$. This makes our result become

$$\left[\frac{dE(\omega)}{dt} \right]_{SC} = \frac{A_H}{8\pi^2} \frac{\omega^4 d\omega}{e^{\beta_H \omega} - 1}. \quad (26)$$

4. String Amplitude

In this section we calculate an emission rate of low energy quanta from a slightly nonextremal state with configuration that 5-brane wound on T^5 and D-string wound on one of the 5-brane direction. We interested in the case that two D-strings annihilate into a graviton. Due to the compactification on T^5 , we can decompose graviton into scalar, vector and graviton in 5-dimensional non-compact spacetime. However, D-string only has a vibration on 5-brane directions. Therefore, the only particle emitted is 5-dimensional scalar.

The 4-point correlation function of D-brane oscillation is given by (Hashimoto and Klebanov, 1996)

$$\begin{aligned} & A(\zeta_1, k_1; \zeta_2, k_2; \zeta_3, k_3; \zeta_4, k_4) : \\ & \int \frac{dx_1 dx_2 dx_3 dx_4}{V_{CKG}} \\ & < \zeta_1 \cdot V_0(2k_1, x_1) \zeta_2 \cdot V_0(2k_2, x_2) \\ & \zeta_3 \cdot V_{-1}(2k_3, x_3) \zeta_4 \cdot V_{-1}(2k_4, x_4) > \end{aligned} \quad (27)$$

where -1 and 0 super ghost picture vertex operator are

$$V_{-1}^{\mu}(z, 2k) = e^{-\phi} \psi^{\mu} e^{i2k \cdot X}(z) \quad (28)$$

$$V_0^{\mu}(z, 2k) = (\partial X^{\mu} + i2k \cdot \psi \psi^{\mu}) e^{2ik \cdot X}(z). \quad (29)$$

Note that we work in unit $\alpha' = 2$ for type I and type II string. To calculate the amplitude, we use contraction from Green function

$$< X^{\mu}(z) X^{\nu}(w) > = -\eta^{\mu\nu} \ln(z - w) \quad (30)$$

$$< \psi^{\mu}(z) \psi^{\nu}(w) > = -\frac{\eta^{\mu\nu}}{z - w} \quad (31)$$

$$< \phi(z) \phi(w) > = -\ln(z - w) \quad (32)$$

The amplitude of 4-point correlation function of type I theory is

$$\begin{aligned} A &= \frac{\Gamma(4k_1 \cdot k_2) \Gamma(4k_1 \cdot k_4)}{\Gamma(1 + 4k_1 \cdot k_2 + 4k_1 \cdot k_4)} \\ & K(\zeta_1, k_1; \zeta_2, k_2; \zeta_3, k_3; \zeta_4, k_4) \end{aligned} \quad (33)$$

where K is kinematic function

$$\begin{aligned} K &= 4k_2 \cdot k_3 k_2 \cdot k_4 \zeta_1 \cdot \zeta_2 \zeta_3 \cdot \zeta_4 \\ &+ 4k_1 \cdot k_2 (\zeta_1 \cdot k_4 \zeta_3 \cdot k_2 \zeta_2 \cdot \zeta_4 \\ &+ \zeta_2 \cdot k_3 \zeta_4 \cdot k_1 \zeta_1 \cdot \zeta_3 \\ &+ \zeta_1 \cdot k_3 \zeta_4 \cdot k_2 \zeta_2 \cdot \zeta_3 \\ &+ \zeta_2 \cdot k_4 \zeta_3 \cdot k_1 \zeta_1 \cdot \zeta_4) \\ &+ \{(1234) \rightarrow (1324)\} \\ &+ \{(1234) \rightarrow (1432)\} \end{aligned} \quad (34)$$

For four world volume photon, we let ζ_I be in the world volume directions. The kinematic function is the same as above. For our calculation, we need scalar scattering, so ζ_I must be in the transverse directions, $\zeta_I \cdot k_m = 0$. This makes the polarization dependent kinematic function become

$$\begin{aligned} K &= 4k_2 \cdot k_3 k_2 \cdot k_4 \zeta_1 \cdot \zeta_2 \zeta_3 \cdot \zeta_4 \\ &+ 4k_2 \cdot k_3 k_3 \cdot k_4 \zeta_1 \cdot \zeta_3 \zeta_2 \cdot \zeta_4 \\ &+ 4k_4 \cdot k_3 k_2 \cdot k_4 \zeta_1 \cdot \zeta_4 \zeta_2 \cdot \zeta_3. \end{aligned} \quad (35)$$

For a case that two open strings scatter into one closed string, graviton, we restrict the momenta p, q in NS sector for open string state and momentum k for closed string state. Only momenta parallel to brane are conserved, $p + q + k_p = 0$. From the conservation of longitudinal momenta, the

only one kinematic invariant variable is $t = 2p \cdot k = 2q \cdot k = -2p \cdot q$. The leading order of amplitude is 2 operators on the boundary and 1 operator on bulk,

$$A = \int \frac{dz_1 dz_2 d^2 z_3}{V_{CKG}} \langle V_1(z_1) V_2(z_2) V_3(z_3, \bar{z}_3) \rangle \quad (36)$$

where

$$V_1(z_1) = \xi_\mu^1 V_0^\mu(z_1, p) \quad (37)$$

$$V_2(z_2) = \xi_\nu^2 V_0^\nu(z_2, q) \quad (38)$$

$$V_3(z_3, \bar{z}_3) = \varepsilon_{\sigma\lambda} V_{-1}^\sigma(z_3, k) \tilde{V}_{-1}^\lambda(\bar{z}_3, k). \quad (39)$$

Note that $\tilde{V}_{-1}^\lambda$ is an antiholomorphic operator of V_{-1}^λ .

We can represent antiholomorphic fields in term of holomorphic fields.

$$\begin{aligned} \tilde{X}^\mu(\bar{z}) &\rightarrow D_\nu^\mu X^\nu(\bar{z}), \\ \tilde{\psi}^\mu(\bar{z}) &\rightarrow D_\nu^\mu \psi^\nu(\bar{z}), \\ \tilde{\phi}(\bar{z}) &\rightarrow \phi(\bar{z}) \end{aligned} \quad (40)$$

D_ν^μ is a diagonal matrix which first $p+1$ entries (longitudinal directions) are equal to 1 and the left $9-p$ entries (transverse directions) are equal to -1. Therefore the 3-point function amplitude is

$$\begin{aligned} A &= \int \frac{dz_1 dz_2 dz_3 d\bar{z}_3}{V_{CKG}} \xi_\mu^1 \xi_\nu^2 \varepsilon_{\sigma\lambda} D_\eta^\lambda \\ &< V_0^\mu(z_1, 2p) V_0^\nu(z_2, 2q) \\ &V_{-1}^\sigma(z_3, k) V_{-1}^\eta(z_4, D \cdot k) \rangle. \end{aligned} \quad (41)$$

We can also write this amplitude as the same with the 4-point correlation function we calculated before by changing some variables,

$$\begin{aligned} 2p &\rightarrow 2k_1 \quad 2q \rightarrow 2k_2 \\ k &\rightarrow 2k_3 \quad D \cdot q \rightarrow 2k_4 \\ \xi_1 &\rightarrow \zeta_1 \quad \xi_2 \rightarrow \zeta_2 \quad \varepsilon \cdot D \rightarrow \zeta_3 \otimes \zeta_4 \end{aligned} \quad (42)$$

For low energy and transverse polarization, we interest decay rate at only specific polarization, $\varepsilon_{67} = 1$, and outgoing graviton has no momentum in string direction, $k_p^1 = 0$, or $p^1 = -q^1$.

$$p \cdot q = p^0 q^0 - p^1 q^1 = 2 |p^1|^2 \quad (43)$$

For low graviton energy, $t \ll 1$, the amplitude is

$$\begin{aligned} A &: \frac{\Gamma(s)\Gamma(t)}{\Gamma(1+s+t)} t^2 \varepsilon_{ij} \xi_1^i \xi_2^j \\ &: \frac{\Gamma(1-2t)}{\Gamma(1-t)^2} t \varepsilon_{ij} \xi_1^i \xi_2^j \rightarrow t \varepsilon_{ij} \xi_1^i \xi_2^j \end{aligned} \quad (44)$$

where s, t, u are Mandelstam variables.

$$s = 4k_1 \cdot k_2 = 4k_3 \cdot k_4 = 4p \cdot q \quad (45)$$

$$t = 4k_1 \cdot k_3 = 4k_2 \cdot k_4 = 2p \cdot k \quad (46)$$

$$u = 4k_1 \cdot k_4 = 4k_2 \cdot k_3 = 2q \cdot k \quad (47)$$

This amplitude coincides with the amplitude obtained from graviton coupled term,

$$\int d^{p+1} x \partial \phi^i \partial \phi^j G_{ij} \quad (48)$$

, in the DBI action (Das and Mathur, 1996)

$$S_{BI} = T \int d^2 \xi e^{-\phi(x)} \sqrt{\det[G_{mn}(x) + B_{mn}(x) + F_{mn}(x)]} \quad (49)$$

$$G_{mn} = G_{\mu\nu}^{(s)}(x) \partial_m X^\mu \partial_n X^\nu \quad (50)$$

$$B_{mn} = B_{\mu\nu}(x) \partial_m X^\mu \partial_n X^\nu \quad (51)$$

We can read out an amplitude of the lowest order interaction between metric fluctuation which 2 open strings scatter into 1 purely transverse graviton,

$$\frac{1}{2} G_{ij} \partial_\alpha X^i \partial^\alpha X^j = \frac{1}{2} (\delta_{ij} + 2\kappa h_{ij}) \partial_\alpha X^i \partial^\alpha X^j \quad (52)$$

One must find an amplitude from the interaction term with specific polarization h_{67}

$$\sqrt{2\kappa} \bar{h}_{67} \partial X^6 \partial X^7; \bar{h}_{67} = \sqrt{2} h_{67} \quad (53)$$

In the calculation, we use the configuration that 5D-brane wrap around T^5 in $(x^5 - x^6 - x^7 - x^8 - x^9)$ directions and D-string wrap around X^5 (which has radius R) Q_1 times with radius of T^4 much smaller than R . Polarization of open strings are in others of 5-brane directions or $(6-7-8-9)$ plane.

The brane decay rate is given by

$$\Gamma(p, q, k) = \frac{\kappa_5^2 (2\pi)^2}{4L} \delta(p_0 + q_0 - k_0) \delta(p_1 + q_1 - k_1) \frac{|A_D|^2}{p_0 q_0 k_0 V_4} \frac{V_4 [d^4]}{(2\pi)^4} \quad (54)$$

where $\kappa_5^2 = \frac{\kappa^2}{2\pi R \tilde{V}_4}$. V_4 is the volume of spatial noncompact directions and $\tilde{V}_4$ is the volume of compact directions.

In order to find a total brane decay rate at a given spectrum, we integrate over momenta of open strings with bosonic string distribution for brane thermodynamics.

$$\Gamma(k) = \int_{-\infty}^{\infty} \frac{L dp_1}{2\pi} \int_{-\infty}^{\infty} \frac{L dq_1}{2\pi} \Gamma(p, q, k) \rho(q_0, q_1) \rho(p_0, p_1) \quad (55)$$

For low energy limit, the decay rate is

$$\Gamma(k) \approx \frac{A_H}{16\pi^4} [d^4 k] \frac{1}{e^{\beta_H k_0} - 1}. \quad (56)$$

Finally, we obtain total energy emission,

$$\frac{dE(k)}{dt} = \frac{A_H}{8\pi^2} \frac{k_0^4 dk_0}{e^{\beta_H k_0} - 1} \quad (57)$$

where $\beta_H = \frac{1}{T_H}$, T_H is the effective temperature of the system which depends on surface gravity. It is the same as the temperature from semiclassical method.

5. Conclusions

In low energy emission, black holes have a thermal radiation of a scalar particle. Photon, fermion and graviton are vanished in the lowest order of calculation. The result shows that energy emission rate calculated from D1D5 system is coincided with the result from classical result. This makes D-brane system a key to study black hole in string theory.

6. References

- Bardeen, JM., Carter, B., and Hawking SW. 1973. The four laws of black hole mechanics. Commun. Math. Phys. (1973)31: 161-170.
- Birrell, ND., and Davies, PCW. 1982. Quantum Fields in Curved Space. New York: Cambridge University Press.
- Das, SR., and Mathur, SD. 1996. Comparing decay rates for black holes and D-branes [Electronic version]. Nucl. Phys. B. 1996(478): 561-576.
- Hashimoto, A., and Klebanov, IR. 1996. Decay of Excited D-branes [Electronic version]. Phys. Lett. B. 1996(381): 437-445.
- Hawking, SW. 1975. Particle creation by black holes. Commun. Math. Phys. (1975)43: 199-220.
- Mukhanov, F., and Winitzki, S. 2007. Introduction to Quantum Fields in Classical Background. New York: Cambridge University Press.