

$$\subseteq ((\Sigma ASA) + \Sigma ASA)$$

$$\subseteq (\Sigma ASA).$$

Hence, S is regular. Therefore, S is both regular and intra-regular.

Theorem 3.12. Let S be an ordered semiring. Then S is both regular and intra-regular if and only if $B = (\Sigma B^2)$ for every ordered bi-ideal B of S .

Proof. It follows from Theorem 3.11 and Lemma 2.3.

Conclusion

The characterizations of regular and intra-regular using ordered quasi-ideals are hold in ordered semiring. Moreover, an ordered semiring S is both regular and intra-regular if and only if $Q = (\Sigma Q^2)$ for every ordered quasi-ideal Q of S .

Acknowledgements

This work has been supported by the Centre of Excellence in Mathematics, the Commission on Higher Education, Thailand.

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