Peaks Over Threshold Model of Generalized Pareto Distributions in Non-life Insurance
การจำลองรูปแบบค่าเกินเกณฑ์ของการแจกแจงพาเรโตวางนัยทั่วไปสำหรับการประกันวินาศภัย

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ABSTRACT

The extreme events are usually modeled by extreme value theory (EVT). A peaks over threshold (POT) model is presented by Generalized Pareto Distribution (GPD) which is considered in the form of exponential and Pareto distribution in this research. The parameters estimations are composed by Pickands, Hill, Decker-Einmahl-de Haan and maximum likelihood estimate (MLE). The retention limit or threshold u of claim severity is analyzed for model fitting in tail distributions. We applied the models to the Danish fire data and motor insurance claims of a Thai non-life insurance company. We have found that the results of the exponential distribution is better fit than Pareto distribution for both Danish fire data and motor insurance claims with threshold u of 8,733,100 Krone and 26,457 Baht, respectively.

บทคัดย่อ

เหตุการณ์ต่าง ๆ ที่มีค่าสุดขีดจะจำลองด้วยทฤษฎีค่าสุดขีดโดยส่วนใหญ่ การจำลองรูปแบบของค่าเกินเกณฑ์จะใช้รูปแบบของการแจกแจงพาเรโตวางนัยทั่วไป ซึ่งในงานวิจัยนี้ได้พิจารณารูปแบบของการแจกแจงเอกซ์โพเนนเชียลและการแจกแจงพาเรโต การประมาณค่าพารามิเตอร์ประกอบด้วย วิธีพิกคานด์ ฮิลล์ เดคเกอร์-อินมาล-เดอฮาน และวิธีภาวะน่าจะเป็นสูงสุด ได้วิเคราะห์ขีดจำกัดความรับผิดหรือค่าเกณฑ์ u ของจำนวนค่าสินไหมทดแทน สำหรับความเหมาะสมสอดคล้องกับรูปแบบที่หางของการแจกแจง ได้ประยุกต์ใช้รูปแบบจำลองดังกล่าวกับข้อมูลไฟไหม้ของประเทศเดนมาร์กและข้อมูลค่าสินไหมทดแทนการประกันภัยรถยนต์ของบริษัทประกันวินาศภัยแห่งหนึ่งในประเทศไทย ผลการศึกษาพบว่า การแจกแจงเอกซ์โพเนนเชียลเป็นการแจกแจงที่มีความเหมาะสมกว่าการแจกแจงแบบพาเรโตสำหรับทั้งสองข้อมูลของข้อมูลไฟไหม้เดนมาร์กและข้อมูลค่าสินไหมทดแทนการประกันภัยรถยนต์ ด้วยค่าเกณฑ์ u เท่ากับ 8,733,100 โครน และ 26,457 บาท ตามลำดับ

Keywords: Extreme value theory, Parameter Estimation, POT

คำสำคัญ: ทฤษฎีค่าสุดขีด ตัวประมาณค่า ค่าเกินเกณฑ์

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Introduction

The extreme value theory (EVT) is mainly represented the modeling of extreme events and providing the model of tail distribution. The extreme events generate extreme value both minimum and maximum which deal with a low frequency of occurrence but consequently the high risk are arisen. The popular methodologies of EVT for applying to financial and insurance risks are the block maxima (minima) (BMM) approach and the peak over threshold (POT) approach. The BMM is based on the Generalized Extreme Value distribution (GEV) which is relevant to distributions of the Gumbel, Fréchet and Weibull distributions. The POT is based on the Generalized Pareto Distribution (GPD) which is considered the exceedances over a given threshold. The EVT is described in a book of Fisher and Tippett (1928), Coles (2001), Kluppelberg (2004), Beirlant *at al.* (2004) and Reiss *at al.* (2007).

The risk of insurance means losses or claims which are defined as the economic losses. The insurer can take some part of insured's losses regarding to a retention limit or a threshold u. The excess of u is a responsibility of a reinsurer. The losses over u is an important data set and it is very useful for reinsurance work. Since the extreme cases are limited for model fitting in a whole data, thus the modeling in tail of distribution is modeled by POT approach. The modeling by POT approach is presented by some papers. For example, Stephenson (2002) proposed the modeling of exceedances over a threshold by maximum likelihood estimation. Anna and Carl (2012) concerned about fitting the GPD to the data on exceedances of high thresholds and presented a tail index estimation by using Hill estimator. Stephaney (2011) proposed an adaptive version of the Hill estimator for parameter estimation and used Monte-Carlo simulations for model illustrations. Rydell (2013) and Vladimir O. *at al* (2012) presented value at risk (VaR) by POT method and solved the model fitting by GPD. Valeria and Matej (2012) presented POT method for modeling of tail distribution with the data of car insurance claims from a Slovak insurance company over the period 1998-2008.

Therefore, we are interested in POT approach for model fitting based on GPD of the claims which are over the threshold u. The mean excess of loss is calculated for expected cost of claims which is very useful for determining of threshold u and pricing in reinsurance.

Objectives of the study

The objectives of the research is are as follows:

1. To find the estimated parameters of GPD for actual data sets.
2. To find a threshold u which is appropriate to the claims data.

Materials and methods

The distribution of GPD is used for modeling exceedances over a threshold u. Their parameters estimation and a statistical test for model fitting are described as follows.

Models

Let a basic losses data $X = X_1, X_2, \dots, X_n$ be a random variables with independent and identically distributed (iid) functions F. We are interested in an estimation of the distribution function F_u which contains some losses above a certain threshold u. Let Y be a random variable which is the exceedance over u such that $Y = X - u$. The distribution of Y is defined as

$$F_Y(y) = P\{X - u \le y | X > u\} \quad ; y > 0$$

$$F_Y(y) = \frac{F_X(y+u) - F_X(u)}{1 - F_X(u)} = \frac{F_X(x) - F_X(u)}{1 - F_X(u)}$$

It is abbreviated as $F_Y(y) = \frac{F(x)-F(u)}{1-F(u)}$.

The F_Y is called the distribution function of exceedances above threshold u. By the conditional probability, F_u can be written by

$$F_u(x) = \begin{cases} \dfrac{F(x) - F(u)}{1 - F(u)} & ; x - u > 0 \\ 0 & ; \ elsewhere. \end{cases}$$

The probability density function f is of the form

$$f_u(x) = \frac{f(x)}{1-F(u)}, \ x - u > 0.$$

For u large enough, the distribution of exceedances above u is approximated by a GPD, i.e.,

$$F_u \approx G_{\xi,\beta}(y) \ as \ u \to \infty \text{ or } f_u \approx g_{\xi,\beta}(y) \ as \ u \to \infty,$$

where $G_{\xi,\beta}(y)$ is the GPD. F_u and f_u are the distribution function and The probability density function of the exceedances above u, respectively.

The Generalized Pareto Distribution (GPD) is defined as follows:

$$G_{\xi,\beta}(y) = \begin{cases} 1 - \left(1 + \xi \dfrac{y}{\beta}\right)^{\frac{-1}{\xi}} & , if \ \xi \ne 0 \\ 1 - \exp\left(\dfrac{-y}{\beta}\right) & , if \ \xi = 0, \end{cases}$$

where $\xi \in \mathbb{R}$ and $\beta > 0$ are the parameters.

In case of $\xi \ne 0$, the distribution is separated into 2 cases. That is, if $\xi \ge 0$, we require $0 \le y < \infty$ and if $\xi < 0$, we require that $0 \le y \le \frac{-\beta}{\xi}$. The parameter ξ is called an extreme value index (EVI). The parameter β is called the scale parameter and it is depending on the threshold u, i.e., $\beta = \beta(u)$.

The probability density function (PDF) of GPD is given by

$$g_{\xi,\beta}(y) = \begin{cases} \dfrac{1}{\beta}\left(1 + \xi \dfrac{y}{\beta}\right)^{\frac{-1}{\xi}-1} & ; \text{if } \ \xi \ne 0 \\ \dfrac{1}{\beta} exp\left(\dfrac{-y}{\beta}\right) & ; \text{if } \ \xi = 0. \end{cases}$$

In our research, the limited distribution of GPD as shown above are considered for 2 distributions which are in case of $\xi = 0$ and $\xi > 0$ for exponential and Pareto distribution, respectively. The models of GPD are as follows.

The exponential distribution

The CDF and PDF are as follows;

$$G_{\xi,\beta}(y) = 1 - exp\left(\frac{-y}{\beta}\right)$$
$$g_{\xi,\beta}(y) = \frac{1}{\beta} exp\left(\frac{-y}{\beta}\right), \quad \text{where } y > 0,\ \beta > 0.$$

The Pareto distribution

The CDF and PDF are as follows;

$$G_{\xi,\beta}(y) = 1 - \left(1 + \xi\frac{y}{\beta}\right)^{\frac{-1}{\xi}}$$
$$g_{\xi,\beta}(y) = \frac{1}{\beta}\left(1 + \xi\frac{y}{\beta}\right)^{\frac{-1}{\xi}-1}, \quad \text{where } y > 0 \text{ and } \xi,\ \beta > 0.$$

Parameters Estimation

The estimated parameters are depending on the models. The β is estimated by maximum likelihood estimate (MLE) for both the distributions of exponential and Pareto. For ξ, we have calculated based on the methods of Hill, Decker-Einmahl-de and Haan, Pickands and MLE. The estimated parameters are compared for model fitting.

The parameters estimations are described as the following items:

Estimating the Shape Parameter ξ

There are 4 methodologies for estimating ξ. The estimated ξ are compared for model fitting.

Define the ordered statistics $X_{i:n}$ as the information i^{th} data for $i = 1,2,,\ldots,n$ which the data is ordered as $X_{1:n} \le X_{2:n} \le \cdots \le X_{n:n}$ pertaining to the original iid random variables $X_1, X_2, \ldots, X_n$. We derived it for $\xi \in \mathbb{R}$. The estimated ξ is calculated under the following estimators.

(1) Hill Estimator

The estimated parameter is of the form

$$\hat{\xi}^{H}_{k,n} = \frac{1}{\mathrm{k}}\sum_{i=1}^{k} \ln(y_{n-i+1:n}) - ln y_{n-k:n},$$

where $k \epsilon \{1,2,3,\ldots,n\}$, $\xi > 0$ and $k > 0$.

(2) Decker-Einmahl-de Haan Estimator

Let $\xi \in \mathbb{R}$ the estimation of ξ is given by

$$\hat{\xi}^{D} = 1 + H_n^{(1)} + \frac{1}{2}\left(\frac{(H_n^{(1)})^2}{(H_n^{(2)})} - 1\right)^{-1},$$

where $H_n^{(1)} = \hat{\xi}^{H}_{k,n}$ and $H_n^{(2)} = \frac{1}{\mathrm{k}}\sum_{i=1}^{k}(\ln(y_{n-i+1:n}) - ln y_{n-k:n})^2$.

(3) Pickands Estimator

The parameter is estimated as

$$\hat{\xi}_{k,n}^{P} = \frac{1}{ln2} ln\left(\frac{y_{n-\left[\frac{k}{4}\right]+1:n} - y_{n-\left[\frac{k}{2}\right]+1:n}}{y_{n-\left[\frac{k}{2}\right]+1:n} - y_{n-k:n}}\right), \quad \text{where } k = \{1,2,\dots,n\}.$$

(4) The Maximum Likelihood Estimate (MLE)

The generic situation is that we observe n-dimensional iid random vector X with probability density function $f(x,\theta)$. It is assumed that the parameter θ is a fixed, likelihood function of θ is defined as

$$L(\theta) = \prod_{i=1}^{n} f(x_i;\theta).$$

The log likelihood function is in the form of

$$lnL(\theta) = \sum_{i=1}^{n} lnf(x_i;\theta).$$

To maximize the natural log of L by first partial derivative with respect to parameter θ is equal to zero, we obtain

$$\frac{\partial}{\partial\theta} lnL(\theta) = 0.$$

We solved the equation as above for estimated parameter $\hat{\theta}$.

Estimation of Parameter β

The β is estimated by MLE such that the description as below.

The exponential distribution

The PDF is in the form of

$$g(y) = \frac{1}{\beta} exp\left(\frac{-y}{\beta}\right), \quad \text{whereas } \beta > 0\,, y > 0.$$

The Likelihood function is the form of

$$L(\beta) = \prod_{i=1}^{k} g(y_i,\beta) \quad = \prod_{i=1}^{k} \frac{1}{\beta} exp\left(\frac{-y_i}{\beta}\right)$$

Take ln, we get that

$$ln\, L(\beta) \;=\; ln \prod_{i=1}^{k} \frac{1}{\beta} exp\left(\frac{-y_i}{\beta}\right) \quad = \sum_{i=1}^{k} ln \frac{1}{\beta} exp\left(\frac{-y_i}{\beta}\right)$$

The maximization by first partial derivative with respect to parameters is equal to zero. They have been shown as below;

$$\frac{d}{d\beta} ln\, L(\beta) = \frac{d}{d\beta}\sum_{i=1}^{k} ln\left(\frac{1}{\beta} exp\left(\frac{-y_i}{\beta}\right)\right)$$
$$= \frac{d}{d\beta}\left[\sum_{i=1}^{k} ln\frac{1}{\beta} + \sum_{i=1}^{k} exp\left(\frac{-y_i}{\beta}\right)\right]$$
$$= \frac{d}{d\beta}\left[\sum_{i=1}^{k}(ln1 - ln\beta) + \sum_{i=1}^{k}\frac{-y_i}{\beta}\right]$$
$$= \frac{d}{d\beta}\left[\sum_{i=1}^{k}(ln1 - nln\beta) - \sum_{i=1}^{k}\frac{y_i}{\beta}\right]$$
$= 0 - \frac{k}{\beta} + \frac{1}{\beta^2}\sum_{i=1}^{k} y_i.$

The estimation for the parameter β can be obtained by solving the equation:

$$\frac{d}{d\beta} L(\beta) = 0.$$

We get

$$\frac{1}{\beta^2}\sum_{i=1}^{k} y_i - \frac{k}{\beta} = 0$$
$$\frac{1}{\beta^2}\sum_{i=1}^{k} y_i = \frac{k}{\beta}$$
$$\frac{1}{\beta}\sum_{i=1}^{k} y_i = k$$
$$\beta = \frac{\sum_{i=1}^{k} y_i}{k}.$$

Therefore $\hat{\beta}$ is an estimated parameter of β. That is $\hat{\beta} = \frac{\sum_{i=1}^{k} y_i}{k}$.

The Pareto distribution

The PDF is in the form of

$$g(y) = \frac{1}{\beta}\left(1 + \xi\frac{y}{\beta}\right)^{\frac{-1}{\xi}-1}, \text{ whereas } \beta > 0, y > 0.$$

The Likelihood function is in the form of

$$L(\xi, \beta) = \prod_{i=1}^{k}\frac{1}{\beta}\left(1 + \frac{\xi y_i}{\beta}\right)^{\frac{-1}{\xi}-1}.$$

Take ln, we obtain

$$lnL(\xi, \beta) = ln\prod_{i=1}^{k}\frac{1}{\beta}\left(1 + \frac{\xi y_i}{\beta}\right)^{\frac{-1}{\xi}-1}$$
$$= \sum_{i=1}^{k}(ln1 - ln\beta) + \sum_{i=1}^{k} ln\left(1 + \frac{\xi y}{\beta}\right)^{\frac{-1}{\xi}-1}$$
$$= \sum_{i=1}^{k}(ln1 - ln\beta) - \left(\frac{1}{\xi} + 1\right)\sum_{i=1}^{k} ln\left(1 + \frac{\xi y_i}{\beta}\right)$$

We get

$$lnL(\xi,\beta) = -k(ln\beta) - \left(1+\frac{1}{\xi}\right)\sum_{i=1}^{k} ln\left(1+\frac{\xi y_i}{\beta}\right)$$

The maximization by first partial derivative with respect to parameters is equal to zero. They have been shown as below;

$$\begin{aligned}\frac{\partial}{\partial\beta}L(\xi,\beta) &= \frac{\partial}{\partial\beta}(-kln\beta) - \left(1+\frac{1}{\xi}\right)\sum_{i=1}^{k}\frac{\partial}{\partial\beta}ln\left(1+\frac{\xi y_i}{\beta}\right)\\ &= \frac{-k}{\beta} - \left(1+\frac{1}{\xi}\right)\sum_{i=1}^{k}\left(\frac{\beta}{\beta+\xi y_i}\right)\left(-\frac{\xi y_i}{\beta^2}\right)\\ &= -\frac{k}{\beta} + \left(\frac{1}{\xi}+1\right)\sum_{i=1}^{k}\frac{\xi y_i}{\beta^2+\beta\xi y_i}.\end{aligned}$$

We get that

$$\frac{\partial L(\xi,\beta)}{\partial\beta} = -\frac{k}{\beta} + \left(\frac{1}{\xi}+1\right)\sum_{i=1}^{k}\frac{\xi y_i}{\beta^2+\beta\xi y_i} = 0.$$

The Newton-Raphson is applied for solving equation to find estimated parameter β.

Goodness of Fit Test

The goodness of fit (GOF) tests measure the compatibility of a random sample with a theoretical probability distribution. We use Kolmogorov-Smirnov Test do decide if a sample come from a hypothesized continuous distribution. It is based on the empirical cumulative distribution function (ECDF). Assume that we have a random sample $y_1, y_2, \dots, y_k$ from some continuous distribution with CDF $G(y)$. The empirical CDF is denoted by

$$G_y^k(y) = \frac{1}{k}[Number\ of\ observations \le y].$$

The theoretical distribution $G_y^*(y)$ and the empirical distribution function $G_y^k(y)$.

The K-S test statistic is defined by

$$D = \max_y |G_y^k(y) - G_y^*(y)|$$

Plots for Threshold

There are 3 criterions for discussion on plots for threshold u, such as mean excess plot, stability of Hill's plot and stability of Pickands plot. The benefit of these are to be guideline for choosing of threshold u.

(1) **Mean Excess Plot**

The mean excess function, $e(u)$, or mean excess over the threshold value *u* is in the form of

$$e(u) = E(X-u|X>u).$$

Since $F_u \approx G_{\xi,\beta}$ thus $e(u)$ is linear function of u. Then we obtain that

$$e(u) = \int_u^\infty \frac{1-F(y)}{1-F(u)}dy = \frac{\beta+\xi u}{1-\xi},$$

for $\beta+\xi u > 0, \xi > 1.$

The mean excess plot (ME plot) is a plotting technique which is plotted between GPD and threshold data.

(2) Stability of Hill's Plot

The Hill's plot is given by points $\left[k, \hat{\xi}_{k,n}^{H}\right]$ where $\hat{\xi}_{k,n}^{H}$ is the Hill estimation.

(3) Stability of Pickands Plot

The Pickands plot is done by points $\left[k, \hat{\xi}_{k,n}^{P}\right]$ where $\hat{\xi}_{k,n}^{P}$ is the Pickands estimation.

Results

The GPD are applied to the actual claims data sets which are composed by fire losses data and motor insurance claims data. Some results are explained as the following items.

Actual Losses Data

Characteristics of Danish Fire Loss Data

The Danish data consist of 2,167 losses over one million Danish Krone (DKK) from the years 1980 to 1990 inclusive. The loss is combined damage of buildings, personal property and loss of profits. The basic characteristic of data is show in Table 1. Figure 1 shows histogram of data.

Table 1 Basic characteristics of data

Count	2,167	Min	1,000,000
55 % Percentile	1,886,300	Max	152,413,200
65 % Percentile	2,259,300	Mean	3,295,900
75 % Percentile	3,021,600	Median	1,774,623
85 % Percentile	4,612,000	Skewness	13.2420
95 % Percentile	8,733,100	Kurtosis	264.6959

Characteristics of Motor Insurance Claims Data

The motor insurance claims data , in Thai Baht, is a voluntary plan which contains 1,296 observations of non-life insurance company in Thailand the year 2009. Figure 2 shows histogram of data

Table 2 Basic characteristics of data

Count	1,296	Min	159
55 % Percentile	8,353	Max	899,879
65 % Percentile	10,789	Mean	17,662
75 % Percentile	16,045	Median	7,296
85 % Percentile	26,457	Skewness	10.6589
95 % Percentile	66,455	Kurtosis	182.8183

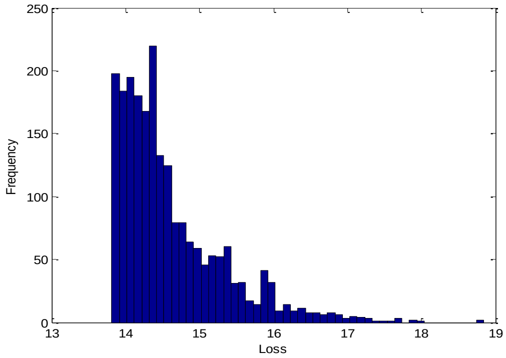

Figure1 Histogram (log scale) of Danish Fire Data

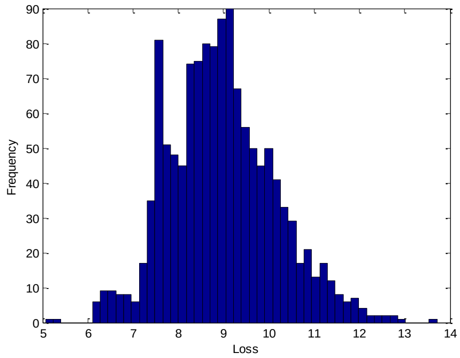

Figure 2 Histogram (log scale) Motor Insurance Claims

Model Fitting Results

The results are contain the D-value of K-S test and estimated parameters that are relied on the threshold u. The number of data, k, is reducing as u increased. The MLE is applied to models, Exponential distribution and Pareto distribution, for $\hat{\beta}$. For pareto distribution, The $\hat{\xi}$ is estimated by MLE and 3 methods of $\hat{\xi}$ estimations.

Danish Fire Data

Table 3 shows the results model fitting and $e(u)$ which are relevant to truncated data based on percentiles. From Table 3, according to K-S test, for all threshold u, the models cannot be fitting to the data sets with a significant level at 0.05. Mostly, D-value of exponential distribution are less than D-value of Pareto distribution which the least value is 0.1184. Pareto distribution, the D-value based on Hill's estimator, are the least. The $e(u)$ of exponential and Pareto distributions trends to be increased as increasing u. At the least D-value at 95th percentile (u = 8,733,100), the $e(u)$ is 12,429,000 Krone. Figure 3 shows the D-value of models with respective to truncated percentiles.

Table 3 GPD fitting to Danish fire data

Item		Value (Percentile) 55	65	85	90	95
$\boldsymbol{u}$		1,886,300	2,259,300	3,021,600	5,920,300	8,733,100
$\boldsymbol{k}$		975	758	542	217	108
Exponential						
	$\hat{\beta}$	3,736,000	4,383,500	5,232,400	8,433,900	12,429,000
	D-value	0.1884	0.1908	0.1896	0.2212	0.1184
	e(u)	3,736,000	4,383,500	5,232,400	8,433,900	12,429,000
Pareto						
MLE	$\hat{\xi}$	0.6244	0.5370	0.6165	0.4431	0.3486
	$\hat{\beta}$	1,594,900	2,116,700	2,948,700	4,718,600	8,016,900
	D-value	0.7975	0.8435	0.7995	0.8908	0.9343
	e(u)	7,382,000	7,192,200	15,102,000	13,184,000	16,980,000
$\hat{\xi}$ estimations						
Hill's	$\hat{\xi}^H_{k,n}$	0.7490	0.7582	0.6311	0.6564	0.6786
	$\hat{\beta}$	1,478,000	1,864,100	2,919,200	4,180,600	6,690,700
	D-value	0.7360	0.7316	0.7919	0.7777	0.7631
	e(u)	11,520,000	14,792,000	18,805,000	23,477,000	39,261,000
Decker	$\hat{\xi}^D_{k,n}$	0.6536	0.6097	0.5895	0.5043	0.4297
de-Haan	$\hat{\beta}$	1,565,200	2,022,200	3,005,200	4,538,500	7,602,000
	D-value	0.7826	0.8049	0.8136	0.8579	0.8937
	e(u)	8,076,500	8,711,500	13,942,000	15,181,000	19,910,000
Pickands	$\hat{\xi}^P_{k,n}$	0.6394	0.5400	0.1040	1.1297	0.0999
	$\hat{\beta}$	1,579,500	2,112,600	5,033,800	3,502,800	10,216,000
	D-value	0.7898	0.8419	0.9969	0.5834	0.9907
	e(u)	7,724,200	7,244,200	6,153,200	-	12,319,000

Motor Insurance Claims

Table 4 shows the results model fitting and $e(u)$ which are relevant to truncated data based on percentiles. From Table 4, according to K-S test, for all threshold u, the models cannot be fitting to the data sets with a significant level at 0.05. Mostly, D-value of exponential distribution are less than D-value of Pareto distribution. Pareto distribution, the D-value based on Hill's estimator, are the least. The $e(u)$ of exponential and Pareto distributions trends to be increased as increasing u. At the least D-value at 85th percentile (u = 33,852), the $e(u)$ is 47,510 Baht. Figure 4 shows the D-value of models with respective to truncated percentile.

Table 4 GPD fitting to Motor insurance claims

Item		Value				
		Percentile				
		55	65	85	90	95
u		8,353	10,789	26,457	36,648	66,455
k		583	454	194	130	65
Exponential						
	$\hat{\beta}$	25,944	30,570	47,510	58,512	75,589
	D-value	0.1926	0.1783	0.1611	0.1408	0.1727
	e(u)	25,944	30,570	47,510	58,512	75,589
Pareto						
MLE	$\hat{\xi}$	0.6589	0.5514	0.5141	0.4241	0.4938
	$\hat{\beta}$	10,722	14,713	24,747	34,495	41,263
	D-value	0.7792	0.8347	0.8522	0.8979	0.8529
	e(u)	47,566	46,057	78,911	86,881	146,360
$\hat{\xi}$ estimations						
Hill's	$\hat{\xi}^{H}_{k,n}$	0.9618	0.9454	0.7480	0.7287	0.5774
	$\hat{\beta}$	9,025	11,979	21,483	28,903	38,974
	D-value	0.6450	0.6506	0.7329	0.7393	0.8081
	e(u)	447,750	406,100	163,780	204,950	183,040
Decker	$\hat{\xi}^{D}_{k,n}$	0.7584	0.6808	0.8535	0.4970	0.4959
de-Haan	$\hat{\beta}$	10,084	13,608	23,631	32,846	41,201
	D-value	0.7310	0.7676	0.8151	0.8588	0.8518
	e(u)	67,947	65,649	93,788	101,500	147,120
Pickands	$\hat{\xi}^{P}_{k,n}$	0.5975	0.5504	0.4227	0.1638	0.6770
	$\hat{\beta}$	11,175	14,723	26,481	43,807	36,665
	D-value	0.8108	0.8353	0.9012	0.9901	0.7570
	e(u)	40,170	45,954	65,249	59,569	252,700

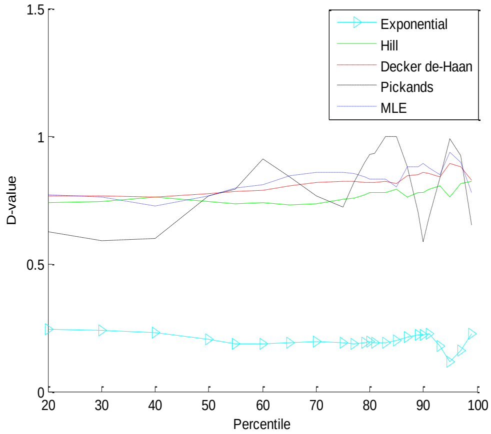

Figure 3 D-value of models based on Danish Fire Data

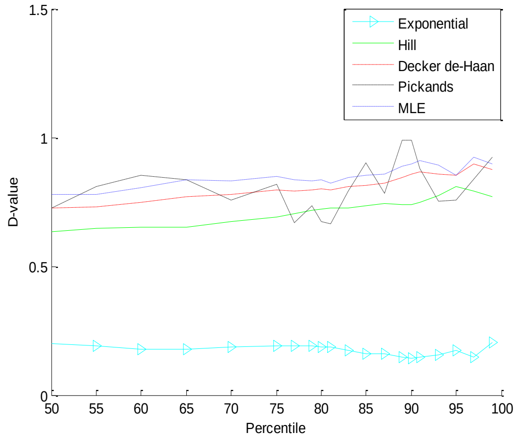

Figure 4 D-value of models based on Motor Insurance Claim

Plots of Threshold

There are $e(u)$ plot, Hill's plot and Pickand's Plot which are the following figure as below.

(1) Danish Fire Data

Figure 5 shows the plot of $\hat{\xi}$ and k. The curve of trend to be straight line when increased. As at $k = 1{,}083$ with $u = 1{,}774{,}623$, the value for all methods are not much difference, i.e., the value $\hat{\xi}$ of Hill, Decker Einmahl-de Haan, Pickands Estimators and MLE are $0.7490, 0.6734, 0.7674$ and 0.6883 respectively.

Figure 6 shows the plot of mean excess of loss and the threshold. The $e(u)$ of all estimation method provided nearly the same value at the threshold $u = 3{,}246{,}200$.

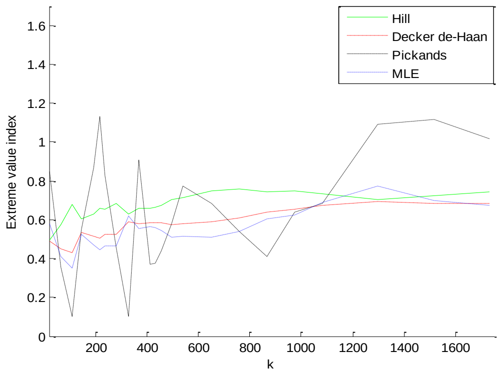

Figure 5 The plot of k and $\hat{\xi}$

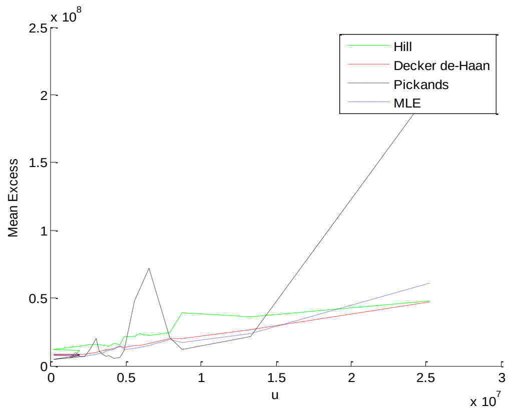

Figure 6 The plot of u and $e(u)$

(2) Motor Insurance Claims

Figure 7 shows the plot of k and $\hat{\xi}$. The curve of $\hat{\xi}$ trend to be straight line when k increased.

Figure 8 shows the plot of mean excess of loss and the threshold u plot based on Motor Insurance Claims. The $e(u)$ of all estimation method provided nearly the same value at the threshold $u = 39{,}986$.

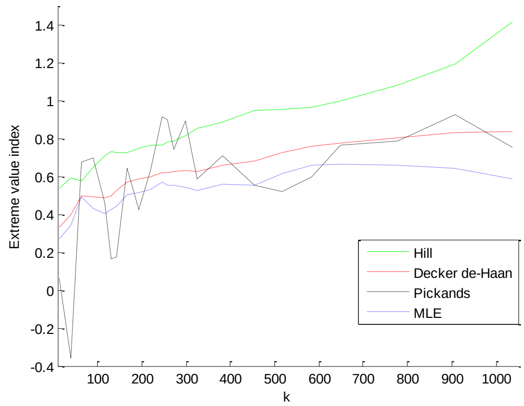

Figure 7 The plot of k and $\hat{\xi}$.

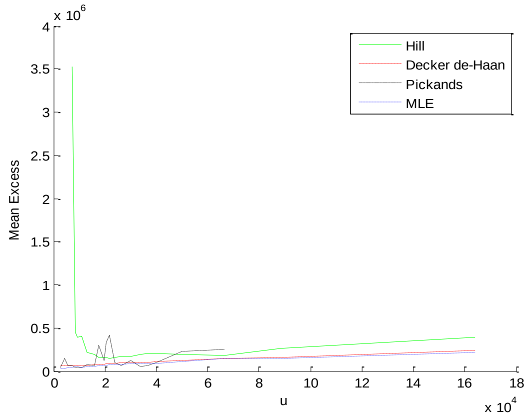

Figure 8 The plot of u and $e(u)$

Discussion and Conclusions

The model of exponential distribution is the better fit to Danish fire data and motor insurance data sets with threshold u are 8,733,100 Krone and 26,457 Baht, respectively.

In this research is pending for analysis of choosing the threshold u which is made optimal of models. It will be continued for further research. Other models that provide more parameters are interesting for study such as the models for 4 and 5 parameters are Kappa Distribution and Wake Distribution, respectively.

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